Decay of Far-Flowfield in Trailing Vortices

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Theme

METHODS for reduction of velocities in trailing vortices of large aircraft are of current interest for the purpose of shortening the waiting time between landings at central airports. We have made finite-difference calculations of the flow in turbulent wake vortices as an aid to interpretation of wind-tunnel and flight experiments directed toward that end. Finite-difference solutions are capable of adding flexibility to such investigations if they are based on an adequate model of turbulence. Interesting developments have been taking place in the knowledge of turbulence that may lead to a complete theory in the future. In the meantime, approximate methods that yield reasonable agreement with experiment are appropriate. The simplified turbulence model we have selected contains features that account for the major effects disclosed by more sophisticated models in which the parameters are not yet established. Several puzzles are thereby resolved that arose in previous theoretical investigations of wake vortices.

Contents

Turbulence model: Since detailed experimental information on the flow in turbulent vortices is not yet available, methods that depend on evaluation of parameters from the particular experiment at hand cannot be used. Instead a complete model is needed that bears an approximate relationship to the exact flow in a manner analogous, for example, to the relationship between the simple kinetic theory and the rigorous kinetic theory of gases. It has been shown by Launder et al.[1] that Prandtl's 1945 turbulent energy model[2] works reasonably well in axisymmetric wakes. The basic equations can be written (in the notation of A. Townsend[3])

$$\frac{D\frac{1}{2}\overline{q^2}}{Dt} = \frac{1}{PR_Q}\nabla\cdot\nu_T\nabla(\tfrac{1}{2}\overline{q^2}) + (a_1)^{1/2}LS^2(\overline{q^2})^{1/2} - a_1^{3/2}(\overline{q^2})^{3/2}/L + F \qquad (1)$$

where

$\frac{1}{2}\overline{q^2}$ = turbulent energy
PR_Q = turbulence Prandtl number (equal to 1.0)
a_1 = Townsend parameter (equal to 0.15)
L = mixing length
S = local shear
F = additional term developed in present paper [Eq. (5)]

$$\nu_T = (a_1)^{1/2}L(\overline{q^2})^{1/2} \qquad (2)$$

In axisymmetric wakes, the mixing length is assigned a uniform value that is taken to be a constant fraction of the width of the wake. However, Launder et al.[1] have developed an additional equation from which the mixing length can be computed rather than assigning its value (again in Townsend's notation)

$$\frac{D\varepsilon}{Dt} = \frac{1}{PR_\varepsilon}\nabla\cdot\nu_T\nabla\varepsilon + C_1\overline{q^2}S^2 - C_2(\overline{q^2})^{1/2}\varepsilon/L \qquad (3)$$

and

$$L = a_1^{3/2}(\overline{q^2})^{3/2}/\varepsilon \qquad (4)$$

where ε = dissipation term in Eq. (1); PR_ε = Prandtl number (equal to 1.3); C_1 = parameter (equal to 0.0643); C_2 = parameter (equal to 0.223). Launder et al.[1] and Hanjalic and Launder[4] have shown that computation of the mixing length in this manner leads to improvement of comparisons with measurements in a variety of turbulent free shear flows as well as boundary layers.

The foregoing turbulence model does not apply to turbulent vortices because it does not account for suppression of turbulence due to curvature of the mean flow that has been disclosed by more advanced models such as that of Donaldson.[5] This effect is illustrated in Fig. 1, which represents a rotating flow in cross section. Rayleigh showed that if the product VR decreases in the outward direction, the flow is unstable against disturbances that would interchange fluid between inner and outer regions. Conversely, if the product VR increases in the outward direction, there is a damping effect on turbulent eddies produced by the shearing motion. This concept can be made quantitative by finding the energy per unit mass of fluid required to interchange fluid between the inner and outer annuli, while retaining an unchanged angular momentum of the fluid transferred

$$\Delta\tfrac{1}{2}\overline{q^2} = \tfrac{1}{2}\Delta(VR)^2\times\Delta 1/R^2 \approx -2(V/R^2)(\partial VR/\partial R)L^2$$

The rate of transfer of fluid by an eddy of size L and velocity $(\overline{q^2})^{1/2}$ is approximated by

$$(1/m)(dm/dt) \approx (\overline{q^2})^{1/2}/L$$

These relations can be combined to obtain an additional term to be included in the turbulent energy Eq. (1). The new turbulence suppression term is

$$F = -2(V/R^2)[\partial(VR)/\partial R]L(\overline{q^2})^{1/2} \qquad (5)$$

The simplified turbulence model described accounts for the major effects disclosed by more advanced models for isotropic shear flows. It therefore applies to turbulent vortices in which axial velocities are small and can be expected to lead to realistic predictions of changes taking place in the far-flow field of wake vortices. We have made finite-difference calculations based on this model in the quasicylindrical approximation using the flow equations described in the full paper.

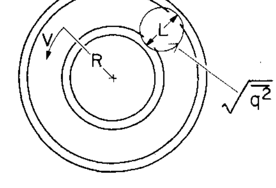

Fig. 1 Suppression of turbulence due to curvature in the mean flow.

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Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Viscous Nonboundary-Layer Flows.

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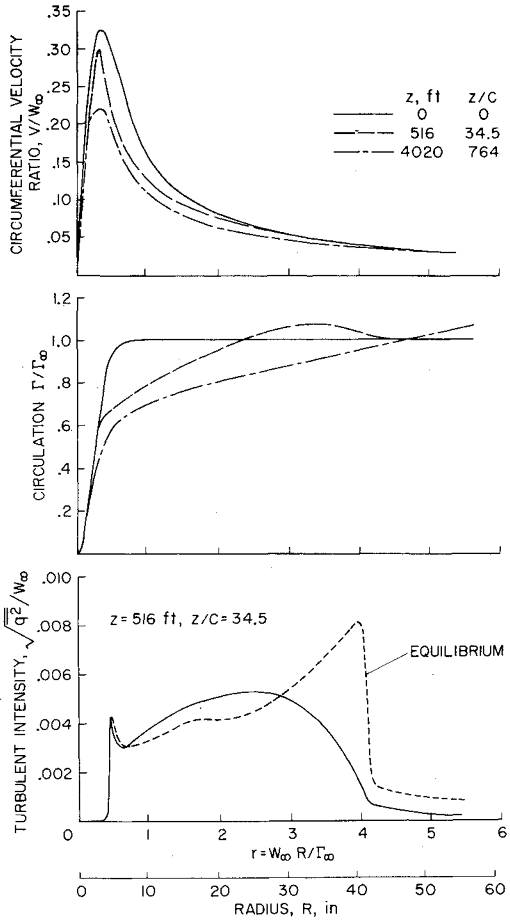

Fig. 2 Profiles in wake vortex ($W_\infty = 132$ fps, $\Gamma_\infty = 112$ ft^2/sec, $C = 5.26$ ft).

Figure 2 shows computed profiles with freestream velocity W_∞, circulation at large radius Γ_∞, and wing chord C assigned the values corresponding to the Cherokee flight data of

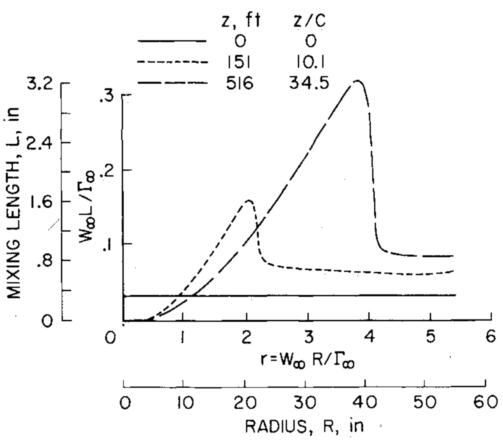

Fig. 3 Mixing length ($W_\infty = 132$ fps, $\Gamma_\infty = 112$ ft^2/sec, $C = 5.26$ ft).

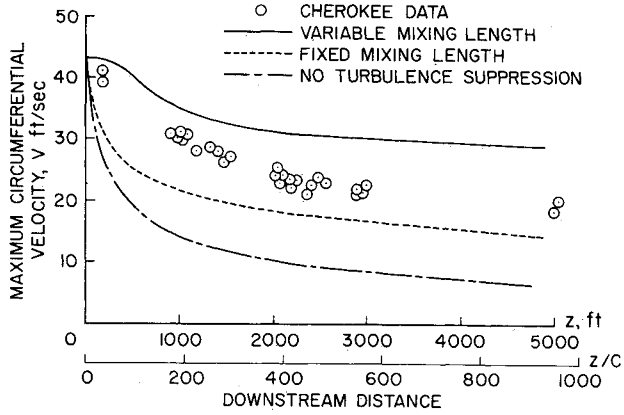

Fig. 4 Decay of wake vortex ($W_\infty = 132$ fps, $\Gamma_\infty = 112$ ft^2/sec, $C = 5.26$ ft).

McCormick, Tangler, and Sherrieb.[6] The calculations were started at $z = 0$ with an initial circulation profile corresponding to that of a self-similar laminar vortex. Figure 3 shows typical mixing length profiles.

Figure 4 indicates the predicted decay of maximum velocity according to several turbulence models. The circles represent the flight data of McCormick et al.[6] The lower curve represents a typical result from previous investigations in which the curvature suppression effect [Eq. (5)] was not accounted for. In the middle curve, the curvature suppression term was included, but a uniform mixing length proportional to the radius of maximum velocity was assigned. In that case the calculations indicate appreciable inward diffusion of turbulence leading to a more rapid decay than indicated by the Cherokee flight data. The upper curve was computed according to the complete simplified turbulence model described in this synoptic. The flight data may be low due to defects in the measurement technique. In any case, this investigation indicates that the level of turbulence inside the radius of maximum velocity is much lower than previously supposed. Two basic mechanisms are responsible for this. One is the suppression of turbulence due to curvature of the flow [Eq. (5)]. The other is the variable mixing length (Fig. 3), which becomes small near the radius of maximum velocity, thereby inhibiting the inward diffusion of turbulence.

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